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spectively. Draw $B'D$ parallel to and of the same currency as BC. Connect G and H. Then prove that $2(AB^2+BC^2+CD^2+DE^2+EA^2)=3CD^2+4(DE.BC.\cos EDB'+EA.AB.\cos EAB)+GH^2$.

Solution by the PROPOSER.

Let a, b, c, and d be the vector sides corresponding with DE, EA, AB, and BC, respectively. Then will $-(a+b+c+d)$ represent the side CD, and $(a-b-c+d)$, the line GH. Squaring each of these six vectors to get their squared tensors, and adding as follows:

$$2[T^2a+T^2b+T^2c+T^2d+T^2.(-a-b-c-d)]$$
$$-3T^2.(-a-b-c-d)-T^2.(a-b-c+d)$$

is found to be $4(Ta.Td.\cos ad+Tb.Tc.\cos bc)$ as required by the problem.*

238. Proposed by O. W. ANTHONY, Head of the Mathematical Department, DeWitt Clinton High School, New York.

Construct a trapezoid having given the sum of the parallel sides, the sum of the diagonals, and the angle formed by the diagonals.

I. Solution by J. H. MEYERS, S. J., Profsssor of Mathematics in Spring Hill College, Mobile, Ala.

Construct the triangle ABC, whose base AB=sum of parallel sides, angle C=angle between diagonals and where $AC+CB$=sum of diagonals. [See Casey's *Sequel to Euclid*, Book III, Prop. 29.]†

Take the point E on AB, such that $CE=CB$; from E draw EF parallel and equal to AC meeting CB in O; join B and F. $CFBE$ is the required trapezoid. In proof, $AE=CF$, hence $EB+CF$=given sum of parallel sides. Since $EF=AC$, $EF+CB$=given sum of parallel sides. And finally, angle $EOB=ACB$.

Analogously solved by J. Scheffer.

II. Remark by G. W. GREENWOOD, M. A., Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill.

Consider any trapezoid $ABCD$ satisfying the given conditions, where AB, CD are the parallel sides. Draw any parallel to AC meeting AB internally in A', say, and hence meeting CD internally in C'. The trapezoid $A'BC'D$ can easily be seen to fulfill the required conditions. Therefore the solution is not unique.

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*In the above solution the lines CB and DE are the ones produced. Prof. G. W. Greenwood has demonstrated that the corresponding theorem, where the lines produced are BC and ED, does not hold true. Ed. E.

†Describe on the same side of AB as a chord the arcs AXB, AYB such that angle AXB=given angle α for every point X, and $AYB=\frac{1}{2}\alpha$. Determine the point P on arc AYB such that AP=sum of diagonals of trapezoid (*i. e.*, sum of sides of the required triangle). Let AP meet arc AXB in C. Then since angle $CPB+CBP=\alpha$, angle $CBP=\frac{1}{2}\alpha$ and $CP=CB$. Therefore ABC is the required triangle. Ed. E.

III. Solution by **G. B. M. ZERR, A. M., Ph. D.,** Parsons, W. Va.

Suppose $ABCD$ the required trapezoid, AB being the shorter (upper) base, $BD=x$, one diagonal, $AC=BF=y$, $BG=$altitude$=h$. Produce DC to F so that $DF=DG+GF=DC+CF=a$, $x+y=c$, $\theta=$given angle. $\sqrt{(x^2-h^2)}+\sqrt{(y^2-h^2)}=a$. $4x^2y^2=a^4+(x^2+y^2)^2+4a^2h^2-2a^2(x^2+y^2)$. But $x^2+y^2=c^2-2xy$. $\therefore 4(c^2-a^2)xy=4a^2h^2+(c^2-a^2)^2$. Now $xy\sin\theta=ah$.

$$\therefore 4a^2h^2-4ah\left(\frac{c^2-a^2}{\sin\theta}\right)=-(c^2-a^2)^2.$$

$$\therefore h=\frac{1}{2a}(c^2-a^2)\tan\tfrac{1}{2}\theta \text{ or } h=\tfrac{1}{2}(c^2-a^2)\cot\tfrac{1}{2}\theta.$$

$$\therefore xy=\frac{c^2-a^2}{4\cos^2\frac{1}{2}a} \text{ or } \frac{c^2-a^2}{\sin^2\frac{1}{2}\theta} \text{ and } x+y=c.$$

$$\therefore x=\tfrac{1}{2}c\pm\frac{\sqrt{(a^2-c^2\cos^2\frac{1}{2}\theta)}}{2\sin\frac{1}{2}\theta} \text{ or } \tfrac{1}{2}c\pm\frac{\sqrt{(a^2-c^2\sin^2\frac{1}{2}\theta)}}{2\cos\frac{1}{2}\theta};$$

$$y=\tfrac{1}{2}c\mp\frac{\sqrt{(a^2-c^2\cos^2\frac{1}{2}\theta)}}{2\sin\frac{1}{2}\theta} \text{ or } \tfrac{1}{2}c\mp\frac{\sqrt{(a^2-c^2\sin^2\frac{1}{2}\theta)}}{2\cos\frac{1}{2}\theta}.$$

Hence lay off $DF=a$, and draw AE parallel to DF at a distance h from it. With D as center and $DB=x$ or y draw DB; then $DF=y$ or x. Draw any line parallel to BF as AC, $A'C'$, $A''C''$, and join B, D to C, C', C'', A, A', A'', respectively. Then any one of the many trapezoids thus formed fulfill the required conditions, as is evident by drawing a figure.

IV. Solution by **J. J. KEYES,** Nashville, Tenn.

Construct the triangle EBF having $BE=$sum of diagonals, $BF=$sum of bases, and angle $E=\frac{1}{2}$ given angle. At F construct the angle $EFD=$angle E, FD meeting BE in D. Through D draw DM parallel to FB. Take any point C on BF, draw CA parallel to FD meeting DM in A. Join AB, DC. $ABCD$ is the required trapezoid. In proof, $AO=CF$. $\therefore AD+BC=BC+BD=$sum of bases. $AC=DF=DE$. $\therefore AC+BD=BD+DE=$sum of diagonals. The angle between BD and $AC=$angle $BDF=2$ angle $E=$given angle.

Also solved by Elmer Schuyler.

CALCULUS.

Problem number 181 was also solved by S. A. Corey and L. C. Walker.

183. Proposed by **W. J. GREENSTREET, A. M.,** Stroud, England.

Evaluate $\displaystyle\int_0^\infty \frac{\sin 2nx dx}{(a^2+x^2)\sin x}$.